Chapter 2:

Fundamentals of Decision

Theory

Decision Theory

- "an analytic and systematic approach to the study of decision making"

Good decisions:

- based on logic

- consider all available data and possible alternatives

- employ a quantitative approach

Bad decisions:

- **not** based on logic

- **do not** consider all available data and possible alternatives

- **do not** employ a quantitative approach

- A good decision may occasionally result in an unexpected outcome; it is still a good decision if made properly

- A bad decision may occasionally result in a good outcome if you are lucky; it is still a bad decision

Steps in Decision Theory

1. **Clearly define the problem at hand**

2. **List the possible alternatives**

3. **Identify the possible outcomes**

4. **List the payoff or profit**

5. **Select one of the decision theory models**

6. **Apply the model and make your decision**

The Thompson Lumber Company

- **Step 1: Clearly define the problem**
  - **The Thompson Lumber Co. must decide whether or not to expand its product line by manufacturing and marketing a new product, backyard storage sheds**

- **Step 2: List the possible alternatives**
  - ***alternative*: "a course of action or strategy that may be chosen by the decision maker"**
    - **(1) Construct a large plant to manufacture the sheds**
    - **(2) Construct a small plant**
    - **(3) Do nothing**

The Thompson Lumber Company

- **Step 3: Identify the outcomes**
  - **(1) The market for storage sheds could be favorable**
    - » **high demand**
  - **(2) The market for storage sheds could be unfavorable**
    - » **low demand**

    state of nature**: "an outcome over which the decision maker has little or no control"**

The Thompson Lumber Company

- **Step 4: List the possible payoffs**
  - **A payoff for all possible combinations of alternatives and states of nature**
  - *Conditional values*: **"payoff depends upon the alternative and the state of nature "**
    - » **with a favorable market:**
      - **a large plant produces a net profit of $200,000**
      - **a small plant produces a net profit of $100,000**
      - **no plant produces a net profit of $0**
    - » **with an unfavorable market:**
      - **a large plant produces a net loss of $180,000**
      - **a small plant produces a net loss of $20,000**
      - **no plant produces a net profit of $0**

Payoff tables

- **A means of organizing a decision situation, including the payoffs from different situations given the possible states of nature**

Decision	States of Nature	
	a	b
1	Payoff 1a	Payoff 1b
2	Payoff 2a	Payoff 2b

  - **Each decision, 1 or 2, results in an outcome, or payoff, for the particular state of nature that occurs in the future**
  - **May be possible to assign probabilities to the states of nature to aid in selecting the best outcome**

The Thompson Lumber Company

	States of Nature	
Decision		

The Thompson Lumber Company

- **Steps 5/6: Select an appropriate model and apply it**

    - **Model selection depends on the operating environment and degree of uncertainty**

Decision Making Environments

- **Decision making under certainty**

- **Decision making under risk**

- **Decision making under uncertainty**

Decision Making Under Certainty

- **Decision makers know with certainty the consequences of every decision alternative**
  - **Always choose the alternative that results in the best possible outcome**

Decision Making Under Risk

- **Decision makers know the probability of occurrence for each possible outcome**
    - **Attempt to maximize the expected payoff**

- **Criteria for decision models in this environment:**
    - **Maximization of expected monetary value**
    - **Minimization of expected loss**

Expected Monetary Value (EMV)

- *EMV*: "the probability weighted sum of possible payoffs for each alternative"
  - Requires a payoff table with conditional payoffs and probability assessments for all states of nature

EMV(alternative i) =   (payoff of 1st state of nature)

X (probability of 1st state of nature)

+ (payoff of 2nd state of nature)

X (probability of 2nd state of nature)

+ . . . + (payoff of last state of nature)

X (probability of last state of nature)

The Thompson Lumber Company

- **Suppose that the probability of a favorable market is exactly the same as the probability of an unfavorable market. Which alternative would give the greatest EMV?**

	States of Nature		
	Favorable Mkt	**Unfavorable Mkt**	
Decision	**p = 0.5**	**p = 0.5**	**EMV**
Large plant	**$200,000**	**-$180,000**	**$10,000**
Small plant	**$100,000**	**-$20,000**	**$40,000**
No plant	**$0**	**$0**	**$0**

EMV(large plant) = (0.5)($200,000) + (0.5)(-$180,000) = $10,000

EMV(small plant) = (0.5)($100,000) + (0.5)(-$-20,000) = $40,000

EMV(no plant) = (0.5)($0) + (0.5)($0) = $0

Build the small plant

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Expected Value of Perfect Information (EVPI)

- **It may be possible to purchase additional information about future events and thus make a better decision**

  - **Thompson Lumber Co. could hire an economist to analyze the economy in order to more accurately determine which economic condition will occur in the future**
    - » **How valuable would this information be?**

EVPI Computation

- **Look first at the decisions under each state of nature**

  - **If information was available that perfectly predicted which state of nature was going to occur, the best decision for that state of nature could be made**

    » *expected value with perfect information* **(EV w/ PI): "the expected or average return if we have perfect information before a decision has to be made"**

EVPI Computation

- **Perfect information changes environment from decision making under risk to decision making with certainty**

  - Build the large plant if you know for sure that a favorable market will prevail

  - Do nothing if you know for sure that an unfavorable market will prevail

Decision	States of Nature	
	Favorable p = 0.5	**Unfavorable** p = 0.5
Large plant	$200,000	-$180,000
Small plant	$100,000	-$20,000
No plant	$0	$0

EVPI Computation

- **Even though perfect information enables Thompson Lumber Co. to make the correct investment decision, each state of nature occurs only a certain portion of the time**

    - **A favorable market occurs 50% of the time and an unfavorable market occurs 50% of the time**

    - **EV w/ PI calculated by choosing the best alternative for each state of nature and multiplying its payoff times the probability of occurrence of the state of nature**

EVPI Computation

EV w/ PI =   (best payoff for 1st state of nature)

X (probability of 1st state of nature)

+ (best payoff for 2nd state of nature)

X (probability of 2nd state of nature)

EV w/ PI =   ($200,000)(0.5) + ($0)(0.5) = $100,000

Decision	States of Nature	
	Favorable p = 0.5	**Unfavorable** p = 0.5
Large plant	**$200,000**	**-$180,000**
Small plant	**$100,000**	**-$20,000**
No plant	**$0**	**$0**

EVPI Computation

- **Thompson Lumber Co. would be foolish to pay more for this information than the extra profit that would be gained from having it**

  - *EVPI*: "the maximum amount a decision maker would pay for additional information resulting in a decision better than one made *without perfect information* "

    » **EVPI is the expected outcome with perfect information minus the expected outcome without perfect information**

EVPI = EV w/ PI - EMV

EVPI = $100,000 - $40,000 = $60,000

Using EVPI

- **EVPI of $60,000 is the <u>maximum</u> amount that Thompson Lumber Co. should pay to purchase perfect information from a source such as an economist**

  - **"Perfect" information is extremely rare**

  - **An investor typically would be willing to pay some amount less than $60,000, depending on how reliable the information is perceived to be**

Opportunity Loss

- **An alternative approach to maximizing EMV is to <u>minimize</u> *expected opportunity loss* (EOL)**

  - *Opportunity loss (regret)* : **"the difference between the optimal payoff and the actual payoff received"**

  - **EOL is computed by constructing an opportunity loss table and computing EOL for each alternative**

Opportunity Loss Table

- **Opportunity loss (regret) for any state of nature is calculated by subtracting each outcome in the column from the best outcome in the same column**

| | States of Nature | | | |
| | Favorable, p = 0.5 | | Unfavorable, p = 0.5 | |
Decision	Payoff	Regret	Payoff	Regret
Large plant	$200,000	$0	-$180,000	$180,000
Small plant	$100,000	$100,000	-$20,000	$20,000
No plant	$0	$200,000	$0	$0
Best payoff	$200,000		$0	

Expected Opportunity Loss

- **Closely related to EMV**
  - *EMV*: "the probability weighted sum of possible payoffs for each alternative"
  - *EOL*: "the probability weighted sum of possible regrets for each alternative"

EOL(alternative i) =   (regret for 1st state of nature)

X (probability of 1st state of nature)

+ (regret for 2nd state of nature)

X (probability of 2nd state of nature)

+ . . . + (regret for last state of nature)

X (probability of last state of nature)

Minimum EOL

	States of Nature		
Decision	Favorable p = 0.5	Unfavorable p = 0.5	EOL
Large plant	$0	$180,000	?
Small plant	$100,000	$20,000	?
No plant	$200,000	$0	?

EOL(large plant) = ?

EOL(small plant) = ?

EOL(no plant) = ?

Summary of Results

Criterion	Decision
Maximize EMV	Build small plant, EMV = $40,000
Minimize EOL	Build small plant, EOL = $60,000

- **Both criteria recommended the same decision**
  - Not a coincidence; these two methods always result in the same decision
  - Repetitious to apply both methods to a decision situation

- **EV w/ PI = $100,000**

  EMV = $ 40,000

  EVPI = $ 60,000 = minimum EOL

Another Example

- **An investor is going to purchase one of three types of real estate: an apartment building, an office building, or a warehouse. The two future states of nature that will determine how much profit the investor will make are either good economic conditions or bad economic conditions. The profits that will result from each decision given these two states of nature are summarized below:**

Purchase Decision	States of Nature	
	Good Economy p = .6	Poor Economy p = .4
Apt Bldg	$ 50,000	$ 30,000
Office Bldg	100,000	-40,000
Warehouse	30,000	10,000

EMV

Purchase Decision	Good Economy p = .6	Poor Economy p = .4	EMV
Apt Bldg	$ 50,000	$ 30,000	?
Office Bldg	100,000	-40,000	?
Warehouse	30,000	10,000	?

EVPI

Purchase Decision	States of Nature	
	Good Economy p = .6	Poor Economy p = .4
Apt Bldg	$ 50,000	$ 30,000
Office Bldg	100,000	-40,000
Warehouse	30,000	10,000

EV w/ PI = ?

EVPI = ?

EOL

Purchase Decision	States of Nature				Expected Opportunity Loss
	Good Economy		Poor Economy		
	p = .6	Regret	p = .4	Regret	
Apt Bldg	$ 50,000		$ 30,000		
Office Bldg	100,000		-40,000		
Warehouse	30,000		10,000		
Maximum					

Decision Making Under Uncertainty

- When probabilities for the possible states of nature can be assessed, EMV or EOL decision criteria are appropriate

- When probabilities for the possible states of nature can not be assessed, or cannot be assessed with confidence, other decision making criteria are required
  - A situation known as *decision making under uncertainty*

- Decision criteria include:
  - Maximax
  - Maximin
  - Equal likelihood
  - Criterion of realism
  - Minimax regret

Maximax Criterion
"Go for the Gold"

- **Select the decision that results in the maximum of the maximum payoffs**

- **A very optimistic decision criterion**
  - **Decision maker assumes that the most favorable state of nature for each decision alternative will occur**

Maximax

Decision	States of Nature		Maximum in Row
	Favorable	Unfavorable	
Large plant	$200,000	-$180,000	**$200,000**
Small plant	$100,000	-$20,000	$100,000
No plant	$0	$0	$0

- **Thompson Lumber Co. assumes that the most favorable state of nature occurs for each decision alternative**

- **Select the maximum payoff for each decision**
  - **All three maximums occur if a favorable economy prevails (a tie in case of no plant)**

- **Select the maximum of the maximums**
  - **Maximum is $200,000; corresponding decision is to build the large plant**
  - **Potential loss of $180,000 is completely ignored**

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Maximin Criterion
"Best of the Worst"

- **Select the decision that results in the maximum of the minimum payoffs**

- **A very pessimistic decision criterion**
    - **Decision maker assumes that the minimum payoff occurs for each decision alternative**
    - **Select the maximum of these minimum payoffs**

Maximin

Decision	States of Nature		Minimum in Row
	Favorable	Unfavorable	
Large plant	$200,000	-$180,000	-$180,000
Small plant	$100,000	-$20,000	-$20,000
No plant	$0	$0	$0

- **Thompson Lumber Co. assumes that the least favorable state of nature occurs for each decision alternative**

- **Select the minimum payoff for each decision**
  - **All three minimums occur if an unfavorable economy prevails (a tie in case of no plant)**

- **Select the maximum of the minimums**
  - **Maximum is $0; corresponding decision is to do nothing**
  - **A conservative decision; largest possible gain, $0, is much less than maximax**

Equal Likelihood Criterion

- **Assumes that all states of nature are equally likely to occur**
  - **Maximax criterion assumed the most favorable state of nature occurs for each decision**
  - **Maximin criterion assumed the least favorable state of nature occurs for each decision**

- **Calculate the *average payoff* for each alternative and select the alternative with the maximum number**
  - ***Average payoff*: the sum of all payoffs divided by the number of states of nature**

- **Select the decision that gives the highest average payoff**

Equal Likelihood

Decision	States of Nature		Row Average
	Favorable	Unfavorable	
Large plant	$200,000	-$180,000	$10,000
Small plant	$100,000	-$20,000	**$40,000**
No plant	$0	$0	$0

Row Averages

$$\text{Large Plant} = \frac{\$200,000 - \$180,000}{2} = \$10,000$$

$$\text{Small Plant} = \frac{\$100,000 - \$20,000}{2} = \$40,000$$

$$\text{Do Nothing} = \frac{\$0 + \$0}{2} = \$0$$

- **Select the decision with the highest weighted value**
  - **Maximum is $40,000; corresponding decision is to build the small plant**

Criterion of Realism

- **Also known as the weighted average or Hurwicz criterion**
  - **A compromise between an optimistic and pessimistic decision**

- **A coefficient of realism, α, is selected by the decision maker to indicate optimism or pessimism about the future**

    $$0 \leq \alpha \leq 1$$

    When α is close to 1, the decision maker is optimistic.
    When α is close to 0, the decision maker is pessimistic.

- ___Criterion of realism___ **= α(row maximum) + (1-α)(row minimum)**
  - **A weighted average where maximum and minimum payoffs are weighted by α and (1 - α) respectively**

Criterion of Realism

- **Assume a coefficient of realism equal to 0.8**

Decision	States of Nature		Criterion of Realism
	Favorable	**Unfavorable**	
Large plant	$200,000	-$180,000	**$124,000**
Small plant	$100,000	-$20,000	$76,000
No plant	$0	$0	$0

Weighted Averages

Large Plant = (0.8)($200,000) + (0.2)(-$180,000) = $124,000

(0.8)($100,000) + (0.2)(-$20,000) = $76,000

Small Plant = (0.8)($0) + (0.2)($0) = $0

Do Nothing =

Select the decision with the highest weighted value

Maximum is $124,000; corresponding decision is to build the large plant

Minimax Regret

- **Choose the alternative that <u>minimizes the maximum regret</u> associated with each alternative**

    - **Start by determining the maximum regret for each alternative**

    - **Pick the alternative with the minimum number**

Minimax Regret

Decision	States of Nature				
	Favorable		Unfavorable		Row
	Payoff	Regret	Payoff	Regret	Maximum
Large plant	$200,000	$0	-$180,000	$180,00 0	$180,00 0
Small plant	$100,000	$100,00 0	-$20,000	$20,00 0	$100,00 0
No plant	$0	$200,00 0	$0	$0	$200,00 0
Best payoff	$200,00 0	0	$ 0		0

- Select the alternative with the lowest maximum regret

Minimum is $100,000; corresponding decision is to build a small plant

Summary of Results

Criterion	Decision
Maximax	Build a large plant
Maximin	Do nothing
Equal likelihood	Build a small plant
Realism	Build a large plant
Minimax regret	Build a small plant

Marginal Analysis

- **Analysis so far has considered decision situations with only a few alternatives and states of nature**
  - **How do we handle situations with a large number of alternatives or states of nature?"**
    - » **a large restaurant is able to stock from 0 to 100 cases of donuts**
      - – **101 possible alternatives = a very large decision table**

- **When marginal profit and loss can be identified, *marginal analysis* can be used as a decision aid instead of a large decision table**

Marginal Analysis

- **Each daily paper stocked by a newspaper distributor costs 19 cents and can be sold for 35 cents. If the paper is not sold by the end of the day, it is completely worthless.**

  - *Marginal profit* **(MP) = "the additional profit made by selling an additional newspaper"**

    MP = 35 cents - 19 cents = 16 cents

  - *Marginal loss* **(ML) = "the loss caused by stocking, but not selling, an additional newspaper"**

    ML = 0 cents - 19 cents = 19 cents

Marginal Analysis

- **Marginal analysis with <u>discrete</u> distributions**
  - – **A manageable number of alternatives/states of nature**
  - – **Probabilities for each state of nature are known**

- **Marginal analysis with the <u>normal</u> distribution**
  - – **A very large number of alternatives/states of nature**
  - – **Probability distribution for the states of nature can be described with a normal distribution**

Marginal Analysis with Discrete Distributions

- To determine the best inventory level to stock:
  - Add an additional unit to a given inventory level only when the expected MP exceeds the expected ML

Let P = the probability that demand $\geq$ a given supply (at least one additional unit is sold)

Let 1 - P = the probability that demand < supply

expected marginal profit = P(MP)
expected marginal loss = (1 - P)(ML)

- The optimal decision rule:
  - Stock an additional unit as long as P(MP) $\geq$ (1 - P)(ML)

$$P \geq \frac{ML}{MP + ML}$$

Marginal Analysis with Discrete Distributions

Solution Process

(1)   Determine the value of P

(2) Construct a probability table, including a cumulative probability column

(3)   Continue ordering inventory as long as $P^* \geq \dfrac{ML}{MP + ML}$

– P^*(the probability of selling one more unit) $\geq$ P

Marginal Analysis with Discrete Distributions

Café du Donut is a New Orleans restaurant specializing in coffee and donuts. The donuts are bought fresh daily and cost $4/carton of two dozen donuts. Each carton of donuts that is sold generates $6 in revenue; any unsold donuts are thrown away at the end of the day. Based on past sales, the following probability distribution applies:

Daily Sales (Cartons)	Probability of Sales at this Level
4	0.05
5	0.15
6	0.15
7	0.20
8	0.25
9	0.10
10	0.10
Total	1.00

Marginal Analysis with Discrete Distributions

(1) Determine the value of P:

$$MP = \$6 - \$4 = \$2; ML = \$0 - \$4 = \$4$$

$$P = \frac{ML}{ML + MP}$$

$$= \frac{\$4}{\$4 + \$2}; P = 0.67$$

(2) Construct a probability table

Daily Sales	Probability	P^* (Probability of Sales This Level or Greater)
4	0.05	1.00
5	0.15	0.95
6	0.15	0.80
7	0.20	0.65
8	0.25	0.45
9	0.10	0.20
10	0.10	0.10
Total	1.00	

Marginal Analysis with Discrete Distributions

Solution Process

(3)   Continue ordering inventory as long as the probability of   selling one more unit is greater than or equal to P

Daily Sales	Probability	P = .67 P (Probability of Sales This Level or Greater)
4	0.05	1.00 $\geq$ 0.67
5	0.15	0.95 $\geq$ 0.67
6	0.15	0.80 $\geq$ 0.67
7	0.20	0.65
8	0.25	0.45
9	0.10	0.20
10	0.10	0.10
Total	1.00	

Marginal Analysis with the Normal Distribution

Four Values are Required

(1)   The average or mean sales for the product, μ

(2) The standard deviation of sales, σ

(3)   The marginal profit for the product

(4)   The marginal loss for the product

Solution Process

(1)   Determine the value of P

(2) Locate P on the normal distribution. For a given area under the curve, find Z from the standard normal table.

$$Z = \frac{X^* - \mu}{\sigma}$$

(3)   Using the relationship,   , solve for X^*, the optimal stocking policy

Marginal Analysis with the Normal Distribution

An Example

Demand for copies of the _Chicago Tribune_ newspaper at Joe's newsstand is normally distributed and has averaged 50 papers per day, with a standard deviation of 10 papers. With a marginal loss of 4 cents and a marginal profit of 6 cents, what daily stocking policy should Joe follow?
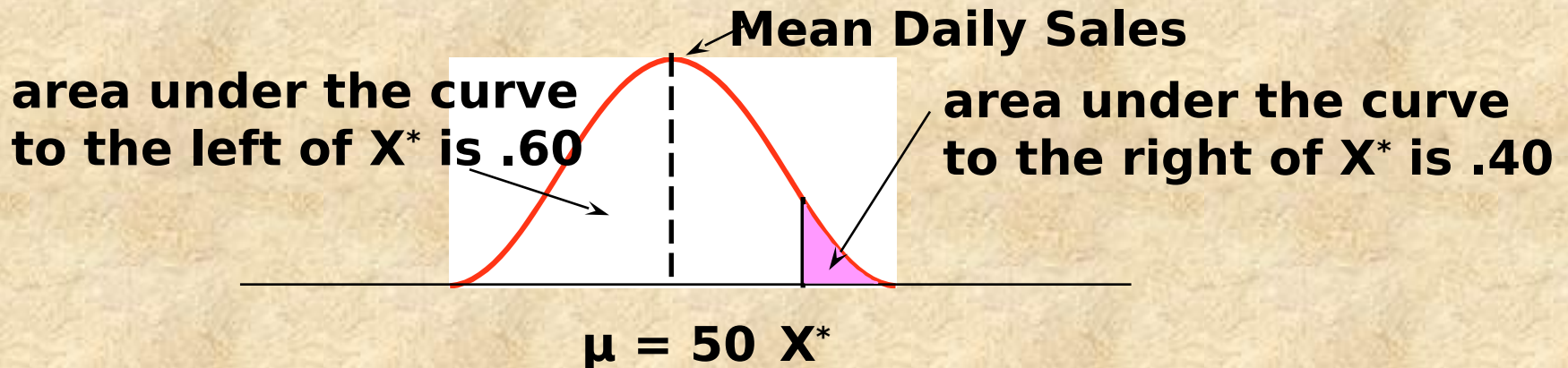
Marginal Analysis with the Normal Distribution

(1) Determine the value of P:

$$\text{MP} = \$.06; \text{ML} = \$.04$$

$$P = \frac{ML}{ML + MP} = \frac{.04}{.04 + .06}; P = 0.40$$

(2) Locate P on the normal distribution; find Z from the standard normal table

Mean Daily Sales

area under the curve to the left of X* is .60

area under the curve to the right of X* is .40

μ = 50 X*

– **Since the normal table has cumulative areas under the curve between the left side and any point, look for 0.60 (= 1.0 - 0.40) to get the corresponding Z value**

Marginal Analysis with the Normal Distribution

(2) **"Find 1 - P in the standard normal table; determine corresponding value of Z"**

APPENDIX A: AREAS UNDER THE STANDARD NORMAL CURVE (p. 806)									
00	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0 0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1 0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2 0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3 0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4 0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5 0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240

Z = 0.25

(3) Using the relationship, $Z = \dfrac{X^* - \mu}{\sigma}$ **, solve for X*, the optimal stocking policy**

$$.25 = \dfrac{X^* - 50}{10}\,; \, X^* = .25(10) + 50 = 53$$

Joe should stock 53 newspapers

Marginal Analysis with the Normal Distribution

When P > 0.5:

"Find P in the standard normal table, determine corresponding value of Z and multiply by -1"

APPENDIX A: AREAS UNDER THE STANDARD NORMAL CURVE (p. 806)									
	00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	**0.79955**	**0.80234**	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891

Z = -0.84

$$-0.84 = \frac{X^* - 100}{10}; \; X^* = -0.84(10) + 100 = 91.6$$

Marginal Analysis with the Normal Distribution

Using Excel

The same four values are required:

(1)   **The average or mean sales for the product, μ**

(2)   **The standard deviation of sales, σ**

(3)   **The marginal profit for the product**

(4)   **The marginal loss for the product**

Solution Process

(1)   **Determine the value of P**

$$\mathbf{P = \frac{ML}{ML + MP} = \frac{.04}{.04 + .06}; P = 0.40}$$

(2)   **Optimal stocking policy = NORMINV(1-P,μ,σ)**

     – NORMINV function returns the inverse of the cumulative normal distribution

Marginal Analysis with the Normal Distribution